MODULE 6.2E

STRUCTURE AND EVOLUTION OF BAROCLINIC WAVES AND FRONTS

Isentropic Potential Vorticity

1. Introduction

Most of the training material presented in Module 6.2 is based on the concepts of diagnosis using quasigeostrophic theory applied on constant pressure surfaces. As we have often discussed, the use of pressure as a vertical coordinate has many practical applications. One of the advantages is that constant pressure surfaces are quasi-horizontal. This facilitates the visualization of meteorological systems in three dimensions.

Another vertical coordinate that can be used in meteorology is potential temperature. This is possible because, in a stable atmosphere, potential temperature increases with height. There are many advantages to viewing the atmosphere on isentropic surfaces. Since potential temperature is conserved in frictionless, adiabatic flow, it is often easier to follow individual parcels of air from one chart to the next. Problems can occur in trying to visualize atmospheric structures using this approach, however, since isentropic surfaces can often be nearly vertical or in some cases can actually fold underneath themselves.

One quantity that we often see analyzed on isentropic surfaces is potential vorticity (PV). PV is a concept with which you are likely familiar from your university studies of dynamic meteorology. For this reason, we will spend only a short time introducing the derivation of PV, and we will try to focus on its practical applications. In its practical application, PV is usually contoured on an isentropic surface, or conversely, we might see charts of potential temperature contoured on a constant PV surface. In either case, this approach of viewing the atmosphere is referred to as "IPV", or isentropic potential vorticity.

2. Vorticity and Potential Vorticity

We are all familiar with the concept of vorticity, and the fact that it is actually the vertical component of the vorticity vector that we consider in operational meteorology. As discussed in Holton (1992, p. 97 ff), the term potential vorticity can refer to a number of concepts, all with slightly different meanings, but all referring to a ratio of vorticity to some vertical depth of the atmosphere. The PV that we see contoured on isentropic surfaces on operational weather maps is the one defined by Ertel (1942):

$$PV \equiv \left(\zeta_\theta + f\right)\left(-g\frac{\partial\theta}{\partial p}\right)$$

(Actually, in the original definition one just sees the letter P, rather than PV, to define Ertel's potential vorticity.)

If we look closely at this definition, we can see that PV is a combination of terms that are all quite familiar to us. First of all, g is simply the acceleration due to gravity, and the minus sign is there so that the value of PV is normally positive in the Northern Hemisphere. The relative vorticity, ζ_θ, in this equation is evaluated on an isentropic surface, and f, as always, is the planetary vorticity. The final term, $\partial\theta/\partial p$, is a measure of the static stability of the atmosphere (we know that in a stable atmosphere, potential temperature increases with height). So, in a sense, we can think of potential vorticity on an isentropic surface as simply the product of absolute vorticity (ζ+f) and static stability.

From the definition of PV given above, the units work out to be $\mathrm{Kkg^{-1}m^{2}s^{-1}}$. For the purposes of contouring on maps, it is convenient to define one PV unit as

$$\mathbf{1\ PVU = 10^{-6}\ Kkg^{-1}m^{2}s^{-1}}$$

There are several reasons why many meteorologists think that the consideration of IPV charts are useful. First of all, PV is a conserved quantity in adiabatic, frictionless flow. The conservation of potential vorticity is a powerful constraint on the large scale motions of the atmosphere. PV centres may be identified on a series of analyses and can be used to describe the evolution of flow patterns during significant synoptic events such as rapid cyclogenesis, blocking and retrogression of longwaves.

Secondly, it is possible to deduce the T, p and wind fields from the PV distribution if a number of assumptions are made. For example, one assumption involves the specification of a balance condition which relates the mass field to the motion field. The simplest balance condition is the quasi-geostrophic approximation. One must also specify an initial reference state and appropriate boundary conditions. Once this is done, however, the spatial distribution of PV then becomes a source term in the equations, the flow field being derived entirely from this term. Later, an analogy will be made with static electric charge distributions and their associated electric fields.

Finally, certain atmospheric processes may be described in terms of the interaction of PV anomalies with the background structure of the atmosphere. For example, when a strong upper-level PV anomaly moves over a low-level baroclinic zone, cyclogenesis usually results. There is no need to invoke secondary circulations (vertical motions) as drivers of the development. In addition, a superposition principle may be used to describe the interaction of PV anomalies at different levels in the atmosphere, interactions which lead to changes in the circulations at these levels.

We will consider each of these points in turn in later sections, and relate these concepts using some sample IPV charts. Before we do that, we will provide a short introduction to the interpretation of the IPV charts.

2. Interpretation of IPV Maps

Figure 1 shows an idealized distribution of potential temperature and potential vorticity over the Northern Hemisphere. This schematic is quite helpful when trying to understand the meaning of various charts of IPV. The figure also illustrates one of the pitfalls of using potential temperature as a vertical coordinate - care must be taken in choosing the isentropic surface. For example, note that the θ = 300 K surface is quite close to sea level in the tropics, but is near the tropopause at 300 mb at the north pole. So, if you were trying to study mid-latitude, middle tropospheric flows, this isentropic surface might be appropriate, but you would need to be aware of these height differences outside of the mid-latitudes.

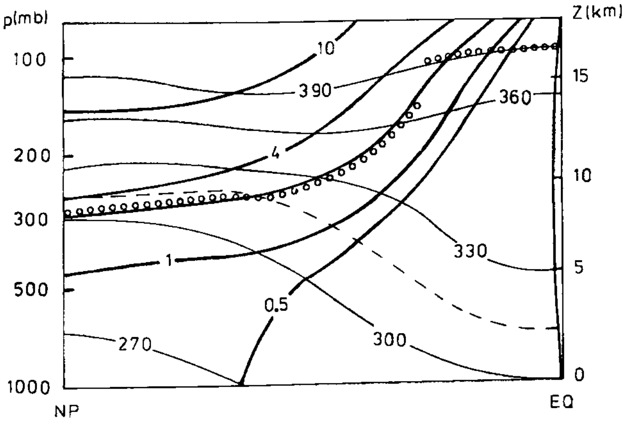

Figure 1. A schematic latitude-height section from the north pole to equator showing isentropes, PV contours and the tropopause. The isentropes are drawn lightly every 30K starting at 270K, but with the 315K surface indicated by dashes. The PV contours are drawn in bold at 0.5, 1, 2, 4, 10 PVU. The tropopause is indicated by a line of open circles. (From Hoskins, 1997)

Many other interesting features that are helpful in interpreting IPV maps can be seen on this schematic:

- PV increases rapidly with height in the stratosphere (because of the high static stability). Because of this, it becomes easy to identify stratospheric air in contrast to tropospheric air on IPV maps.
- The value of PV at the tropopause in mid-latitudes is about 2 PVU. In fact. Some researchers define the tropopause surface to be 1.5 PVU. Many IPV maps consist of potential temperature contoured on the tropopause, as defined in PVU.
- In the tropics, the tropopause is difficult to define, and in the case of this schematic, it is defined using potential temperature rather than potential vorticity.
- In many of the maps that are available to operations, you will see PV contoured on an isentropic surface at $\theta = 315$ K or $\theta = 330$ K. Note that in the middle latitudes, these θ surfaces will consist of both tropospheric and stratospheric air.

Now, let us look at PV from the viewpoints of conservation, invertibility, and cyclogenesis.

3. PV Conservation

The conservation properties of PV have been recognized for a number of years. Rossby (1940) showed that absolute vorticity is approximately conserved following the motion in 2-dimensional horizontal flow. This fact was contained in his barotropic description of large and synoptic scale dynamics. The streamfunction for the flow is derived by inverting the Laplacian operator which relates it to the vorticity:

$$\zeta_a = f + \zeta_r = f + \nabla^2 \Psi$$

where ψ is the streamfunction.

Rossby also noted that the quantity ς_a/h was conserved following the motion in a barotropic model which allows vertical motion, h being the depth of a material fluid column. According to this model, vorticity is produced by a stretching of the material column in the vertical, so-called vortex tube stretching. This process is simply the law of conservation of angular momentum applied to a column of fluid. As the tube stretches, the column shrinks in the horizontal and the average angular velocity of individual fluid elements about the vertical axis of rotation increases, resulting in an increase of relative vorticity. This familiar picture has been used to describe the horizontal deflection of a low-level zonal flow over a mountain range.

Ertel derived a more general expression for PV conservation. Assuming θ is a function of p and ρ alone, then Ertel's theorem (for 3-dimensional, non-hydrostatic, adiabatic flow) is:

$$PV = \frac{1}{\rho}\zeta_a \bullet \nabla_3 \theta = \text{constant}$$

following the motion. This expression is the 3-dimensional analogue of Rossby's conservation principle for 2-dimensional barotropic flow. It is possible to generalize this expression even further to include frictional and diabatic effects.

In the early 1940's, PV was employed as an atmospheric tracer, owing to its conservation properties. Observational studies in the 1950's demonstrated that stratospheric air could be advected down to lower levels within frontal zones. This process was dubbed tropopause folding, a term which is discussed elsewhere in Module 6.2 of the internship course.

Another use of PV as a tracer is through the construction of maps showing the distribution of PV on particular isentropic surfaces. An example of a series of such maps is shown in Figure 2. The isentropic surface is carefully chosen to highlight the stratospheric air descending into the lower troposphere. This air is associated with a developing cutoff low over the North Atlantic. The high PV air in the cutoff appears to be pinched off from the primary reservoir, which is the familiar arctic vortex. The fact that we can follow the PV centre associated with the new cyclone for a period of four days is an illustration of conservation of PV in the real atmosphere.

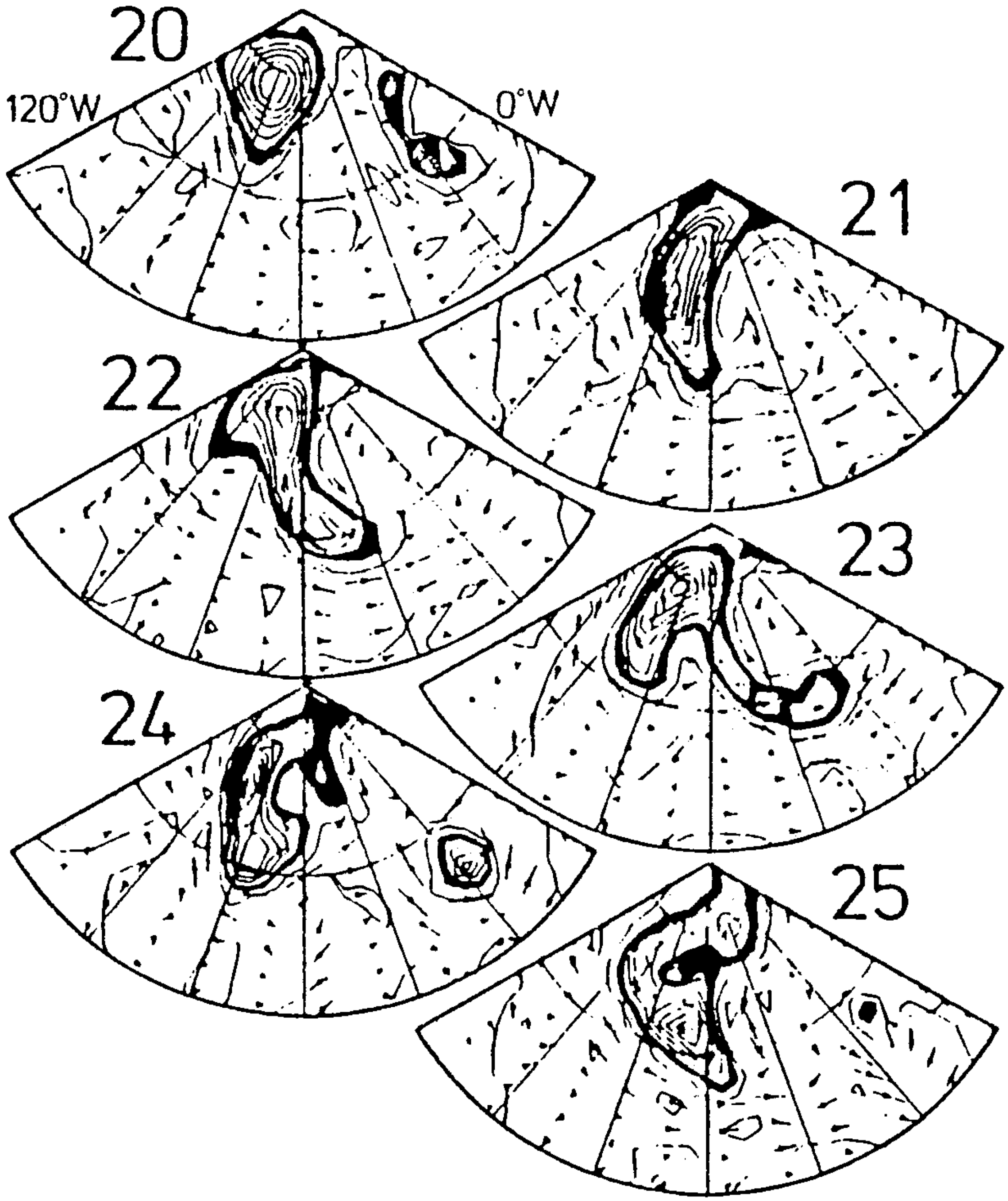

Figure 2. Sectors of the 300 K IPV maps for the period 20-25 September 1982. The region covered is from 40°N to the north pole and from 120°W to 0°W with the 60°W meridian central. The contour interval is 0.5 PV units and the region 1.5-2 units is blacked in. Also shown are the horizontal velocity vectors on this surface (from Hoskins et al, 1985).

4. Invertibility

Pioneering work on the relation between PV anomalies in the upper levels and cyclogenesis was done by Kleinschmidt in the 1950's. He was the first to state the invertibility principle mentioned earlier, which states that a knowledge of the spatial distribution of PV is sufficient to determine the overall structure of the flow.

For example, a positive PV anomaly on an isentropic surface (an IPV anomaly) near the tropopause will induce a cyclonic circulation. This circulation is strongest near the level of the anomaly and weakens below this level. An example of a symmetric circulation about an upper-level IPV anomaly is shown in Figure 3.

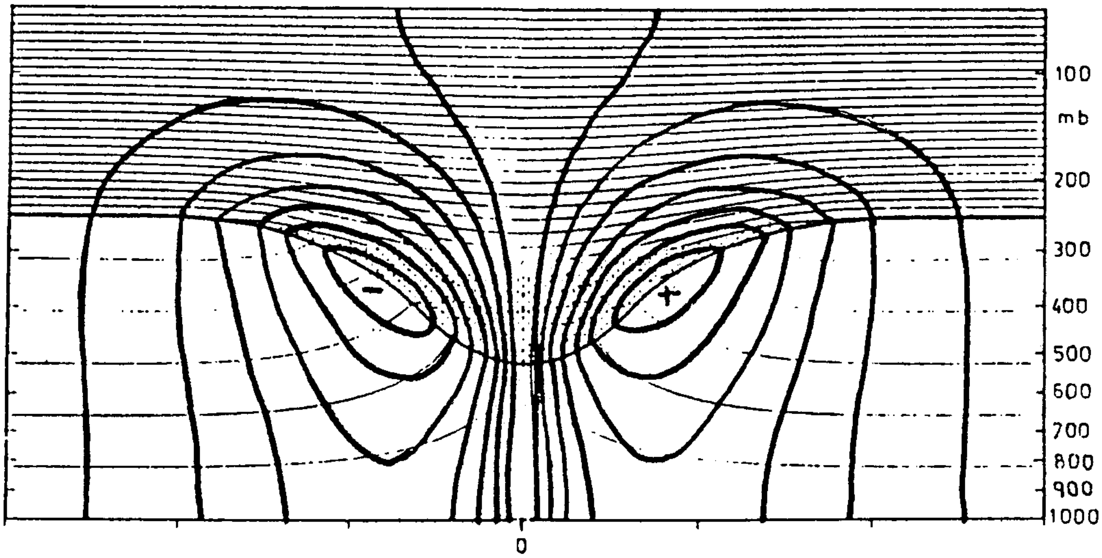

Figure 3. The symmetric circulation about an upper-level positive IPV anomaly. Potential temperature contours are the quasi-horizontal ones. The circulation which is induced by the anomaly is indicated by the. heavier contours.

5. Cyclogenesis

Synopticians are familiar with the strong correlation between the passage of a positive vorticity centre in the upper levels over a preexisting low level baroclinic zone and the incidence of cyclogenesis. In terms of IPV thinking, this type of development may be understood as follows (see Fig. 4 taken from Hoskins et al., 1985).

The positive IPV anomaly at upper levels has a strong cyclonic circulation associated with it. As noted earlier, a weaker extension of this circulation extends down to the surface. As the anomaly moves over the baroclinic zone, this low level circulation induces a wave in the thermal field, the wave crest forming a positive temperature anomaly. It is also true that a positive temperature anomaly is equivalent to a positive IPV anomaly. This new centre establishes its own cyclonic circulation, the upwards extension of which can eventually reinforce the flow about the upper centre. Thus, a process of mutual reinforcement (positive feedback) is established.

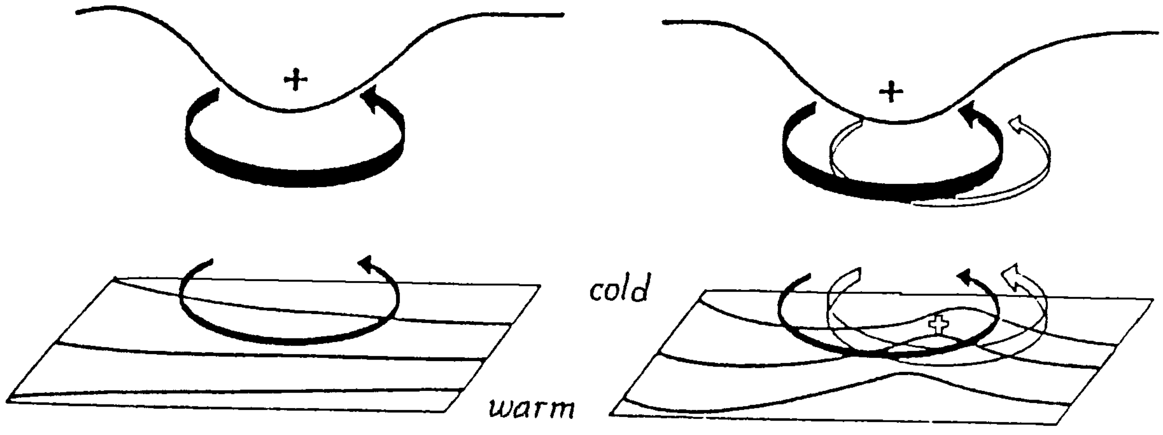

Figure 4. A schematic picture of cyclogenesis associated with the arrival of an upper air IPV anomaly over a low level baroclinic region. On the left, the upper air cyclonic IPV anomaly, indicated by a solid plus sign and associated with the low tropopause shown, has just arrived over a region of significant low level baroclinicity. The circulation induced by the anomaly is indicated by solid arrows, and potential temperature contours are shown on the ground. The low level circulation is shown above the ground for clarity. The advection by this circulation leads to a warm temperature anomaly somewhat ahead of the upper IPV anomaly as indicated on the right, and marked with an open plus sign. This warm anomaly induces the cyclonic circulation indicated by the open arrows. If the equatorward motion at upper levels advects high-PV polar lower-stratospheric air , and the poleward motion advects low-PV subtropical upper-tropospheric air, then the action of the upper-level circulation induced by the surface potential temperature anomaly will, in effect, reinforce the upper air IPV anomaly and slow down its eastward progression.

The circulations induced by both centres are combined to yield the net circulation. If the quasi-geostrophic approximation is used, then the two (or more) fields may be superimposed to arrive at the net flow. A useful comparison is made between this description and the superposition principle in electrostatics. The IPV centres take the place of the electric charge distributions and the induced circulations are analogous to the resulting electric fields. When more than one IPV anomaly is present, the net flow is derived by calculating the vector sum of all of the individual flow fields. Also, as in the electrostatic case, the reference state and boundary conditions must be specified in order to arrive at a meaningful solution.

This suggests that interactions between IPV centres may result in "feedbacks" which either enhance or dissipate extratropical cyclones. This interaction is illustrated in the computer animation of the rapidly deepening east coast cyclone (the President's Day storm of 1979) which is available for viewing from our library.

Another example of this kind of interaction is shown in Fig. 5 (Bleck 1990). The growth of a surface cyclone over the U.S. Gulf States is illustrated. Absolute vorticity is plotted in isentropic coordinates, which is equivalent to PV plotted in pressure coordinates. At first, note the westward displacement of the upper vorticity centre relative to the surface cyclone. Note also the growth of the upper and lower regions of vorticity as the upper centre approaches the lower one. Eventually, the positive feedback between the two causes them to link in the vertical.

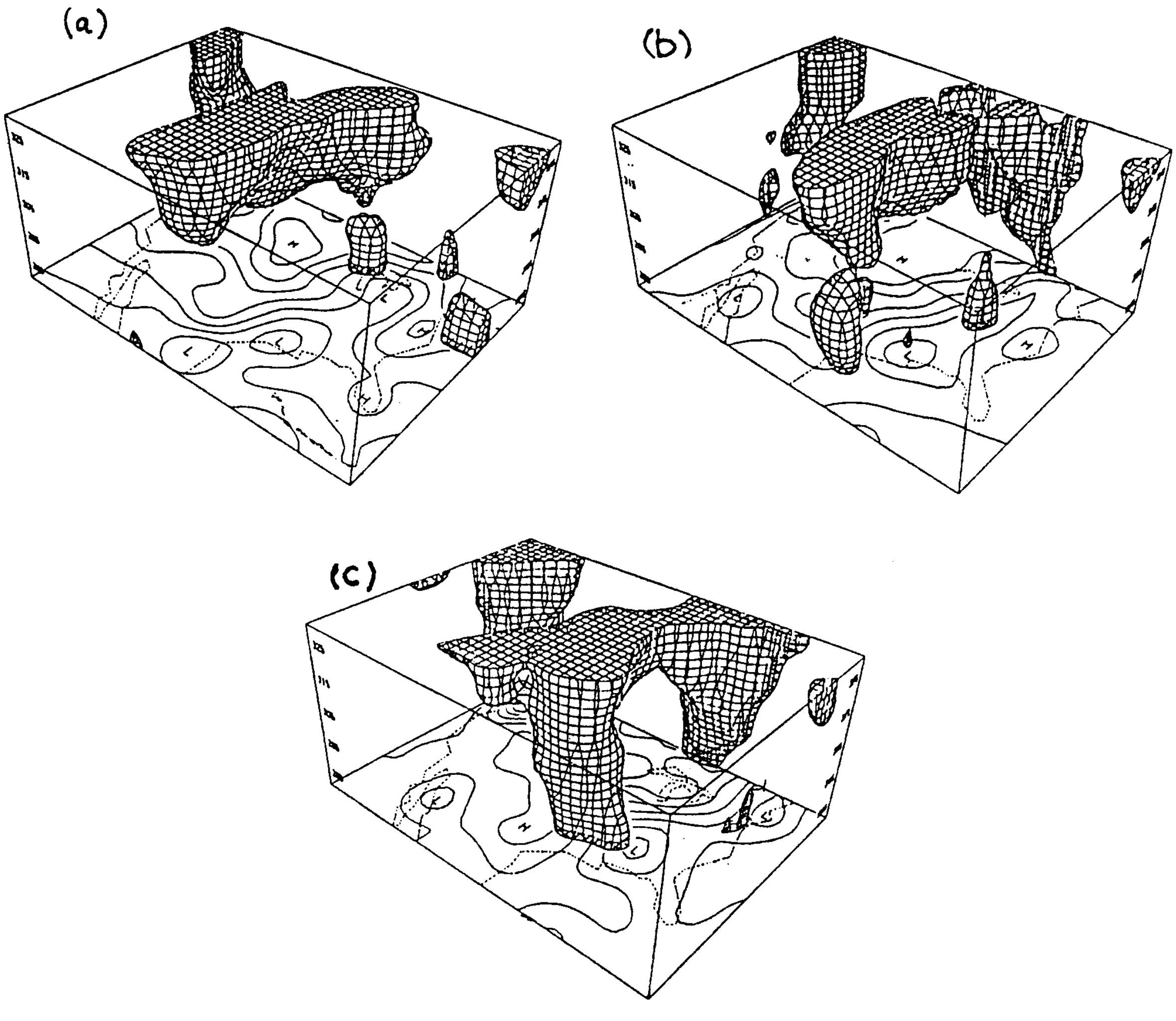

Figure 5. Iso-surfaces of absolute vorticity in (x, y, θ) space over North America at 0000 and 1200 UTC 3 March 1989. Vorticity threshold value 15 x 10^{-5}. Sea level isobars plotted on bottom (contour interval 4 mb). Ordinate: potential temperature ranging from 280 to 330 K.

These perspective plots of PV in 3-dimensions depict the growth of the mid-latitude cyclone at all levels. This may be compared with the current operational practice of plotting absolute vorticity only at 500 mb and then using quasi-geostrophic theory to infer vertical motion and explain development. Clearly, the 3-dimensional plots provide a more complete description of the changes in vorticity throughout the troposphere as the surface cyclone evolves. However, it remains to be seen whether such depictions are operationally useful.

6. Further Discussion

In this paper, a lot of emphasis has been placed on the conservation of PV and the importance that this principle has in the diagnosis of large scale flows. However, it is possible for PV to be created or destroyed when the flow is not adiabatic and frictionless. Low level PV anomalies are eroded by friction. In the free atmosphere, the diabatic heating effect can be very important in the material change of PV. Above a region of latent heat release the PV is decreased and below it the PV is increased. Thus, the latent heating in a mid-latitude low pressure system can lead to significant enhancement of the low level cyclonic circulation, and the enhancement of the upper ridge in the air moving ahead of it.

The potential vorticity approach to looking at atmospheric patterns has many advocates. It is currently most popular with those doing research in meteorology. For operational purposes, it should be remembered that this approach is neither more nor less accurate than the “conventional” quasigeostrophic diagnosis on pressure coordinates, it merely provides another perspective. For this reason, there may be those who prefer to look at conventional isobaric charts, and those who prefer to see IPV charts (or perhaps both) while doing diagnosis. However, one should at least be familiar with the principles behind PV and uses of IPV charts in order to remain current with the science of meteorology.

7. References

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